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Unpolarized Gluon Distribution Functions in Leading Order and Next-to-Leading Order

Dr. Ranjit Baishya
Department of Physics, J. N. College, Boko, Assam, India – 781123

The unpolarized gluon distribution functions have been obtained by solving Dokshitzer-Gribove-Lipatov-Alterelli-Parisi (DGLAP) evolution equations in LO and NLO at the small-x limit. Here we have used a Taylor series expansion and then the method of characteristics to solve the evolution equations. We have also calculated t and x-evolutions of gluon distribution functions and the results are compared with GRV1998 [1] and MRST2004 [2] gluon parameterizations.

I. THEORY

The DGLAP evolution equations in standard forms for unpolarized gluon distribution functions in LO and NLO [3-10] are

$$\frac{\partial G(x,t)}{\partial t} - \frac{\alpha_S}{2\pi}\frac{2}{3}\left\{\left(\frac{11}{12} - \frac{N_f}{18} + \ln(1-x)\right)G(x,t) + I_g^1\right\} = 0, \qquad (1)$$

$$\frac{\partial G(x,t)}{\partial t} - \frac{\alpha_S}{2\pi}\frac{2}{3}\left\{\left(\frac{11}{12} - \frac{N_f}{18} + \ln(1-x)\right)G(x,t) + I_g^1\right\} - \left(\frac{\alpha_S}{2\pi}\right)^2 I_g^2 = 0, \qquad (2)$$

where functions $I_g^1(x,t)$ and $I_g^2(x,t)$ are defined in Appendix F.

Let us introduce the variable u = 1□ ω and using Taylor's expansion series we can rewrite

$$G\left(\frac{x}{\omega},t\right) = G(x,t) + \frac{xu}{1-u}\frac{\partial G(x,t)}{\partial x} + \frac{1}{2}\left(\frac{xu}{1-u}\right)^2\frac{\partial^2 G(x,t)}{\partial x^2} + \cdots\cdots$$

$$\approx G(x,t) + \frac{xu}{1-u}\frac{\partial G(x,t)}{\partial x}, \qquad (3a)$$

$$F_2^S\left(\frac{x}{\omega},t\right) = F_2^S(x,t) + \frac{xu}{1-u}\frac{\partial F_2^S(x,t)}{\partial x}. \qquad (3b)$$

Since x is small in our region of discussion, the terms containing x^2 and higher powers of x are neglected. Using equations (3a) and (3b) and performing u-integrations we get equation (1) of the form

$$\frac{\partial G(x,t)}{\partial t} - \frac{A_f}{t}\left[A_1^g(x)G(x,t) + A_2^g(x)\frac{\partial G(x,t)}{\partial x} + A_3^g(x)F_2^S(x,t) + A_4^g(x)\frac{\partial F_2^S(x,t)}{\partial x}\right] = 0, \qquad (4)$$

where $A_f = \frac{\alpha_S(t)}{3\pi}t = \frac{4}{3\beta_0} = \frac{4}{33-2N_f}$, as defined in chapter 3, and also

$$A_1^g(x) = -\frac{11}{6} + 2x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \ln(x), \qquad A_2^g(x) = 1 + \frac{4}{3}x - 3x^2 + x^3 - \frac{1}{4}x^4 + 2x\ln(x),$$

$$A_3^g(x) = \frac{2}{9}\left\{-\frac{3}{2} + 2x - \frac{1}{2}x^2 - 2\ln(x)\right\}, A_4^g(x) = \frac{2}{9}\left\{2 + \frac{1}{2}x - 3x^2 + \frac{1}{2}x^3 + 4x\ln(x)\right\}.$$

Now let us assume, $F_2^S(x,t)=R(x)G(x,t)$, where R(x) is a suitable function of x or may be a constant. Now equation (8.4) gives

$$-t\frac{\partial G(x,t)}{\partial t}+A_f L_1^g(x)\frac{\partial G(x,t)}{\partial x}+A_f M_1^g(x)G(x,t)=0, \quad (5)$$

where

$$L_1^g(x)=A_2^g(x)+R(x)A_4^g(x) \text{ and } M_1^g(x)=A_1^g(x)+R(x)A_3^g(x)+A_4^g(x)\frac{\partial R(x)}{\partial x}$$

Now let us consider two new variables S and τ instead of x and t, such that

$$\frac{dt}{dS}=-t, \quad (6a)$$

$$\frac{dx}{dS}=A_f L_1^g(x). \quad (6b)$$

Putting these in equation (5), we get

$$\frac{dG(S,\tau)}{dS}+M_1^g(S,\tau)G(S,\tau)=0, \quad (7)$$

where $M_1^g(S,\tau)=A_f M_1^g(x)$. Equations (7) can be solved as,

$$G(S,\tau)=G(0,\tau)\exp\left[-\int_0^S M_1^g(S,\tau)dS\right] \quad (8)$$

For t-evolution, gluon distribution function varies with t remaining x constant. Hence equation (6a) can be used to solve the equation (7). Now we have to replace the co-ordinate system (S, τ) to (x, t), considering when S = 0, t = t_0 and the input function as $G(\tau)=G(x,t_0)$. So the t-evolution of gluon distribution function in LO is given by

$$G(x,t)=G(x,t_0)\left(\frac{t}{t_0}\right)^{A_f M_1^g(x)}, \quad (9)$$

Using equation (6b) and replacing the co-ordinate system (S, τ) to (x, t), with consideration when τ = 0, $x=x_0$ and the input function as $G(S)=G(x_0,t)$, the x-evolution of gluon distribution function in LO is given by

$$G(x,t)=G(x_0,t)\exp\left(-\int_{x_0}^{x}\frac{M_1^g(x)}{L_1^g(x)}dx\right) \quad (10)$$

Similarly the t and x-evolution of gluon distribution functions in NLO are given by

$$G(x,t)=G(x_0,t)\left(\frac{t}{t_0}\right)^{A_f\left\{M_1^g(x)+T_0 M_2^g(x)\right\}}, \quad (11)$$

$$G(x,t)=G(x_0,t)\exp\left(-\int_{x_0}^{x}\frac{M_1^g(x)+T_0 M_2^g(x)}{L_1^g(x)+T_0 L_2^g(x)}dx\right), \quad (12)$$

where

$$L_2^g(x)=B_2^g(x)+R(x)B_4^g(x) \text{ and } M_2^g(x)=B_1^g(x)+R(x)B_3^g(x)+B_4^g(x)\frac{\partial R(x)}{\partial x}.$$

We also consider $\left(\frac{\alpha_S(t)}{2\pi}\right)^2 = T_0\left(\frac{\alpha_S(t)}{2\pi}\right)$, where T_0 is a numerical parameter, not arbitrary choose but determined by phenomenological analysis [79, 174]. The other functions are $B_1^g(x) = -\frac{52}{3}\ln x$, $B_2^g(x) = -\frac{52}{3}(1 - x + x\ln x)$, $B_3^g(x) = \int_x^1 A(\omega)\,d\omega$ and $B_4^g(x) = x\int_x^1 \frac{1-\omega}{\omega} A(\omega)d\omega$.

II. RESULT AND DISCUSSIONS

Here we have compared our result of t-evolution for gluon distribution function G(x, t) in LO and NLO with GRV1998 global parameterizations [177] and x-evolution with GRV1998 and MRST2004 [178] parameterizations. We consider GRV1998 parameterization for $10^{\square\square\square} \le x \le 10^{\square\square 5}$ and $20 \le Q^2 \le 40\ GeV^2$, where they used H1 [182] and ZEUS [183] high precision data on G(x, Q^2). They have chosen $\alpha_S(M_Z^2) = 0.114$ and $\Lambda_{\overline{MS}}(N_f = 4) = 246$ MeV. The input densities have been fixed using the data sets HERA [182], SLAC[184], BCDMS [185], NMC [113] and E665 [114]. The resulting input distribution at $Q^2 = 0.04\ GeV^2$ is given by $xg = 20.80\, x^{1.6}(1-x)^{4.1}$.

We have taken the MRST2004 fit to the H1 [53] and ZEUS [54] data with x < 0.01 and $2 \le Q^2 \le 500\ GeV^2$ for $Q^2 = 100\ GeV^2$, in which they have taken parametric form for the starting distribution at $Q_0^2 = 1 GeV^2$ given by

$$xg = A_g x^{-\lambda_g}(1-x)^{3.7}\left(1+\varepsilon_g\sqrt{x}+\gamma_g x\right) - Ax^{-\delta}(1-x)^{10},$$

where power of the (1□x) factors are taken from MRST2001 fit [56]. Here $A_g, \lambda_g, \gamma_g, \varepsilon_g$, A and δ are taken as free parameters. The optimum fit corresponds to $\alpha_S(M_Z^2) = 0.119$ and $\Lambda_{\overline{MS}} = 323$ MeV with $N_f = 4$.

Our results represent the best fit graphs of our work with different parameterization curves. Results of parameterization at lowest-Q^2 values are taken as input to test the t-evolution equations and those at highest-x is taken as input to test the x-evolution equations. We have compared our results for R(x) as a constant R, a power function ax^b and an exponential function $ce^{\square dx}$. In our work for gluon distribution function, we have found the values of the gluon distribution function remains almost same for b<0.0001 and for d<0.001. So, we have chosen b = 0.0001 and d = 0.001 and the best fit graphs are observed by changing the values of R, a and c. If we plot $T^2(t)$ and $T_0T(t)$ against Q^2, then we can see that for T_0 = 0.048, the values of T^2 and $T.T_0$ are nearly same in our region of discussion, as we have seen in figure 4.1 of chapter 4. Thus we consider T_0 = 0.048 in calculation of G(x, t) at NLO and the consideration of parameter T_0 does not give any abrupt change in our results.

In figures 8.1, we have plotted our results of t-evolution of gluon distribution function in LO from equation (9) and compared with GRV1998 parameterization for R(x) = R, a constant. Here we have plotted our results of gluon distribution function against Q^2 for x = $10^{\square 5}$ and x = $10^{\square 4}$ and we get the best fit with R = 0.5 and R = 0.8 respectively.

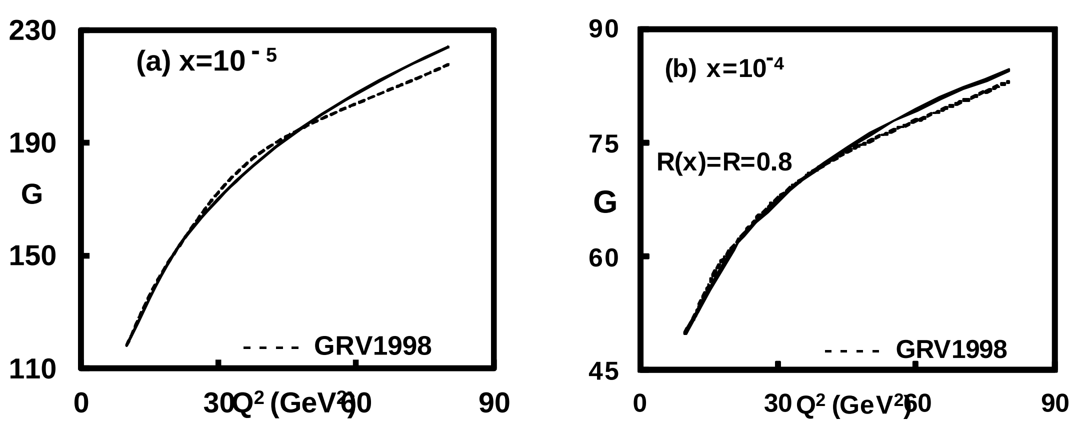

Figure 1: t-evolution of gluon distribution functions in LO for R(x) = R, a constant, compared with GRV1998 parameterization graphs

In figures 2, we have plotted our results of t-evolution of gluon distribution function in LO from equation (9) and compared with GRV1998 parameterization at $x=10^{-5}$ for $R(x) = ax^b$ and ce^{-dx} respectively. Here we have plotted our results against Q^2 for $R(x)= ax^b$ as well as $R(x) = ce^{-dx}$ and we get the best fit with a = 0.9, b = 0.0001 and c = 0.8, d = 0.001.

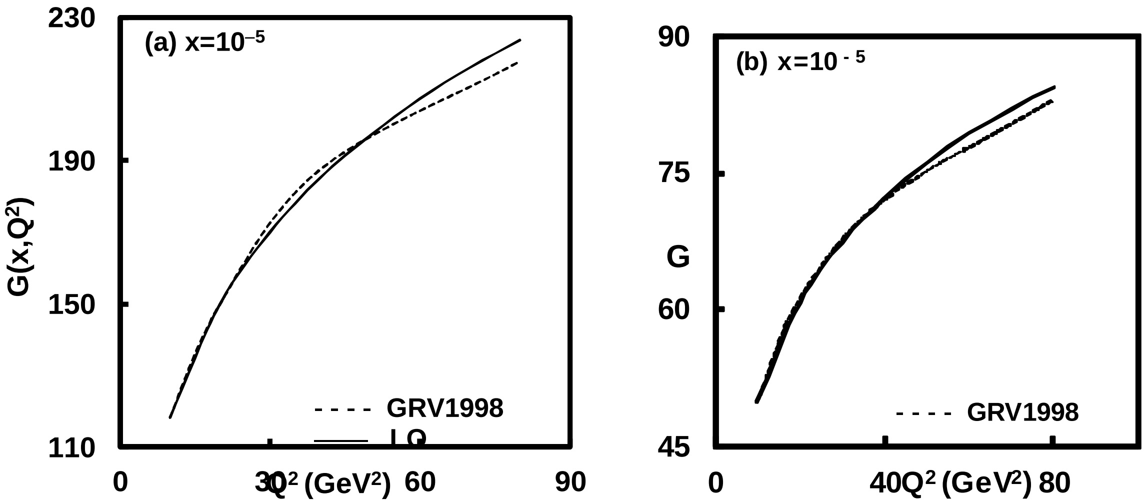

Figure 2: t-evolution of gluon distribution functions in LO for $R(x)= ax^b$ and ce^{-dx} compared with GRV1998 parameterization graphs

In figure 3 we have plotted our results of gluon distribution function against x for Q^2= 100 GeV^2 with R(x) = R and compared with MRST2004 and best fit has been found for R= 0.4.

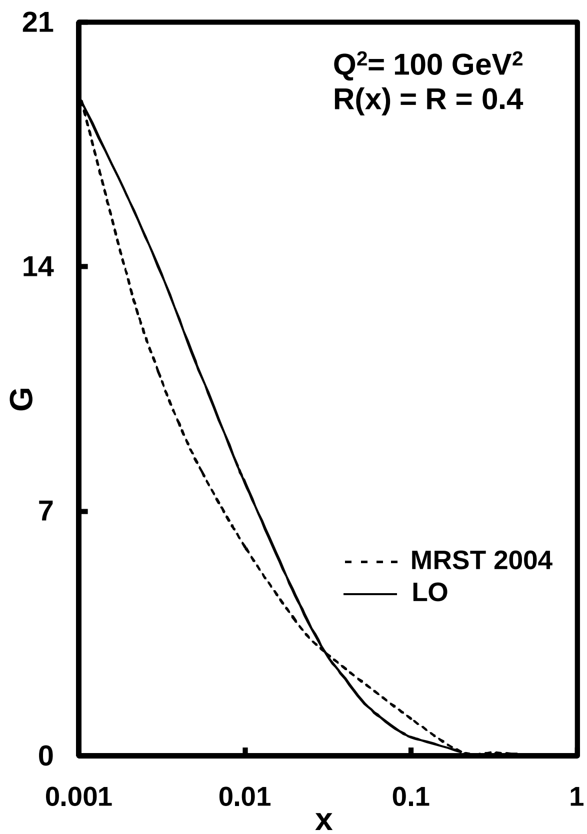

Figure 3: x-evolution of gluon distribution functions in LO for R(x) = R, compared with MRST2004 parameterization

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In figures 4 and 5, we have plotted our results of t-evolutions of gluon distribution function in NLO from equation (11) and compared with GRV1998 gluon parameterization for R(x) = R and ax^b. These results are also compared with our LO results obtained from equation (9). In figure 4, we have plotted our results of gluon distribution function against Q^2 for $x = 10^{-5}$ and we get the best fit with R = 0.8 in NLO and R = 0.9 in LO.

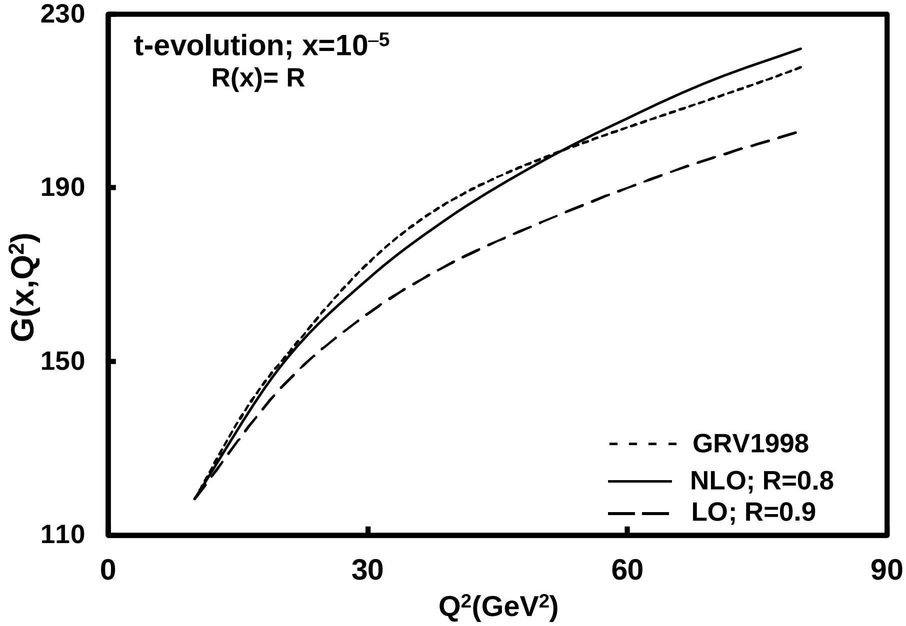


Figure 4: t-evolution of gluon distribution functions in NLO for R(x) = R, compared with GRV1998 parameterization

In figures 5 our results of gluon distribution function in NLO have been plotted against Q^2 for $x = 10^{-4}$ with $R(x) = ax^b$ and we get the best fit with a = 0.8 and b = 0.001 in NLO and a = 0.9 and b = 0.0001 in LO.

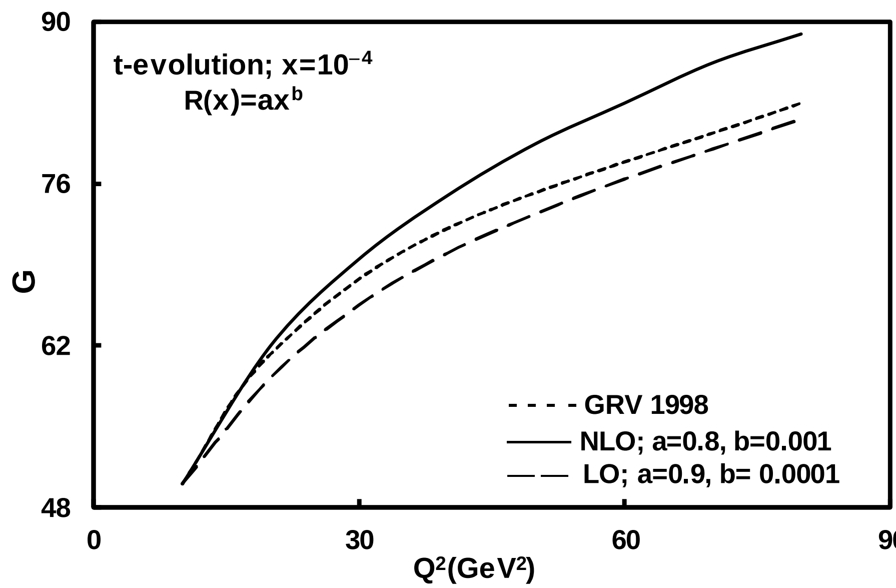
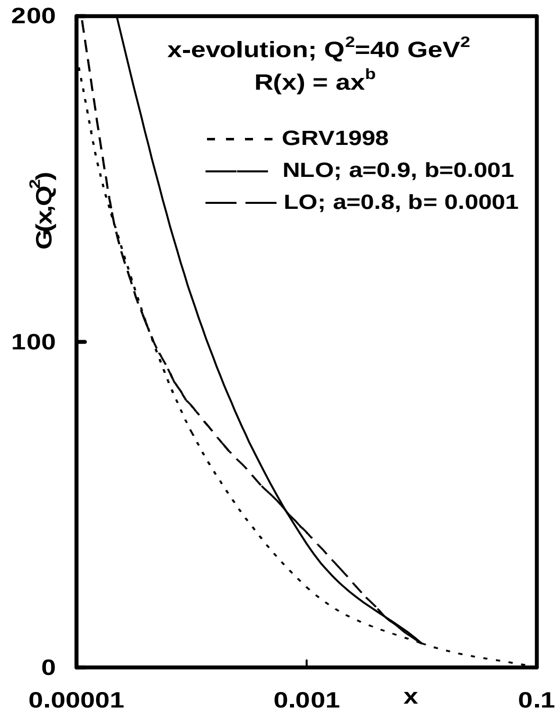


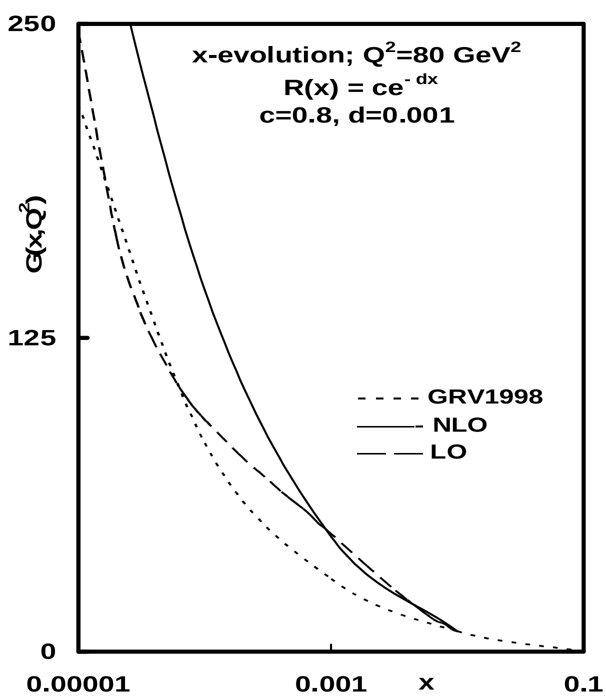
Figure 5: t-evolution of gluon distribution functions in NLO for $R(x) = ax^b$, compared with GRV1998 parameterization

In figure 6, we have plotted our results of x-evolution of gluon distribution function in NLO from equation (12) and compared with GRV1998 gluon parameterization for $R(x) = ax^b$. These results are also compared with our LO results. We have plotted our results for Q^2= 40 GeV^2 and the best fit has found for a = 0.9 and b = 0.001 in NLO and for a = 0.8 and b = 0.0001 in LO.



Figures 6: x-evolution of gluon distribution functions in NLO for $R(x)= ax^b$, compared with GRV1998 parameterization

In figure 7, we have plotted our results of x-evolution of gluon distribution function in NLO from equation (12) and compared with GRV1998 gluon parameterization for $R(x) = ce^{-dx}$. These results are also compared with our LO results. We have plotted our results for Q^2= 80 GeV^2 and the best fit has found with c = 0.8 and d = 0.001 in both NLO and LO.



Figures 7: x-evolution of gluon distribution functions in NLO for $R(x)= ce^{-dx}$, compared with GRV1998 parameterization

In figure 8 we have plotted our results of gluon distribution function against x for Q^2=100 GeV^2 with R(x) = R, a constant and R(x) = ax^b, a power function of x. Our results are compared with MRST2004 as well as our LO results. The best fit has been found with R= 0.4 in both NLO and LO and also with a = 0.5, b = 0.0001 in NLO and a = 0.5, b = 0.001 in LO respectively.

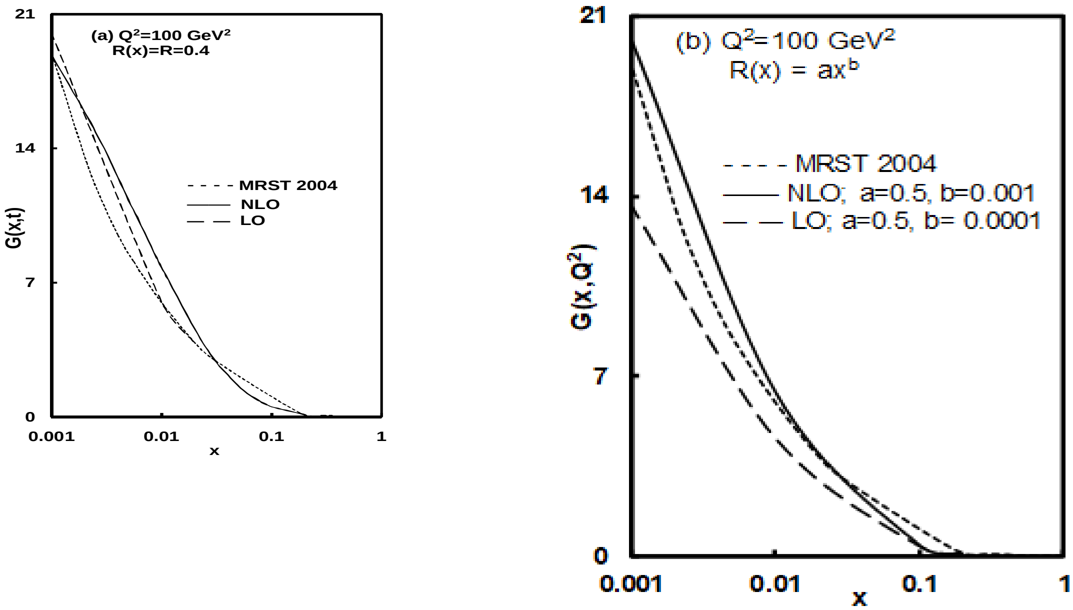


Figure 8: x-evolution of gluon distribution functions in NLO for R(x) = R, a constant and ax^b, compared with MRST2004 parameterization

III. CONCLUSION

In this chapter, we have solved the DGLAP evolution equation by using method of characteristics and get gluon distribution function in LO and NLO. We have calculated here the t and x-evolutions of gluon distribution functions. It is shown that our results are in good agreement with GRV1998 and MRST2004 global parameterizations especially at small-x and high-Q^2 region. Here from global parameterizations and our results we have seen that the gluon distribution functions increase when x decreases and Q^2 increases for fixed values of Q^2 and x respectively. On an average, the mean percentage errors of our LO and NLO results are 7.73% and 1.86% with GRV1998 and also 11.45% and 3.63% with MRST2004 global parameterizations respectively. These errors of our results are very less as compared to systematic and statistical uncertainties in the experimental data. Thus, there is significant contribution of Next–to–Leading order over the Leading order in unpolarized gluon distribution functions.

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