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ORIGINAL PAPER

Solution of singlet Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation in next-to-next-to-leading order at small-x

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Abstract: In this paper we have obtained the singlet structure function by solving the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equation in next-to-next-to-leading order at the small-x limit. Here we have used a Taylor series expansion to solve the evolution equations to get the t (=ln Q^2/Λ^2) and x-evolutions of structure functions, where x is the Bjorken variable, Q^2 is the four momentum transfer in a deep inelastic scattering (DIS) process and Λ is the QCD cut off parameter. We have also calculated the t and x-evolutions of deuteron structure functions. Results are compared with recent experimental data and parametrizations.

Keywords: DGLAP equation; Deep inelastic scattering (DIS); Structure function

PACS Nos.: 12.38.-t; 12.39.-x; 13.60.Hb

1. Introduction

Deep inelastic scattering (DIS) establishes the scale evolution of the structure functions which is one of the most important precision tests of perturbative quantum chromodynamics (QCD) [1]. It provides unique information about the deep structure of the hadrons. Most importantly, measurements of these structure functions in DIS allow extracting the parton densities, which are subsequently used as inputs for many other hard scattering processes. Recently a considerable amount of theoretical and experimental effort has been reported in the accurate determination of the parton distributions of the nucleon, and in particular their associated errors in view of the accurate determination of QCD parameters, test of sum rules and computation of collider processes [2]. The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations [3–6] are the fundamental tools to study the t (=ln Q^2/Λ^2) and x-evolutions of structure functions. Hence the solutions of DGLAP evolution equations give quark and gluon distribution functions which ultimately produce proton, neutron and deuteron structure functions.

The solutions of the unpolarized DGLAP equation for the QCD evolution of structure functions have been discussed considerably over the past years [7–16]. In a recent paper, the non-singlet structure function at small-x in next-to-next-to-leading order (NNLO) has been obtained from DGLAP equation by using Taylor expansion method [17]. In the present paper, we would intend to solve the DGLAP equation analytically by Taylor expansion method to obtain a semi-numerical solution for singlet structure functions at small-x in NNLO. Inclusion of the NNLO contributions considerably reduces the theoretical uncertainty of determinations of the quark and gluon densities from deep-inelastic structure functions. Recently the three-loop splitting functions are introduced with excellent phenomenological success [18–20]. Here, we have studied the impact of the NNLO contributions on the evolution of the singlet quark and gluon densities and on the most important singlet structure function, $F_2^S(x,t)$. We have also calculated t and x-evolutions of deuteron structure functions and results are compared with Fermilab E665 Collaboration [21] and New Muon Collaboration (NMC) [22] experimental data and Neural Network Parton Distribution Functions (NNPDF) [23] parametrizations.

2. Theory

The DGLAP evolution equation for singlet structure function in NNLO is [24, 25]

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$$\frac{\partial F_2^S}{\partial t} - \frac{\alpha_S(t)}{2\pi}\left[\frac{2}{3}\{3+4\ln(1-x)\}F_2^S(x,t) + I_1^S(x,t)\right] - \left(\frac{\alpha_S(t)}{2\pi}\right)^2 I_2^S(x,t) - \left(\frac{\alpha_S(t)}{2\pi}\right)^3 I_3^S(x,t) = 0, \quad (1)$$

where

$$I_1^S(x,t) = \frac{4}{3}\int_x^1 \frac{d\omega}{1-\omega}\left[(1+\omega^2)F_2^S\left(\frac{x}{\omega},t\right) - 2F_2^S(x,t)\right] + N_f\int_x^1 \left\{\omega^2+(1-\omega)^2\right\}G\left(\frac{x}{\omega},t\right)d\omega,$$

$$I_2^S = \left[(x-1)F_2^S(x,t)\int_0^1 f(\omega)d\omega + \int_x^1 f(\omega)F_2^S\left(\frac{x}{\omega},t\right)d\omega + \int_x^1 F_{qq}^S(\omega)F_2^S\left(\frac{x}{\omega},t\right)d\omega + \int_x^1 F_{qg}^S(\omega)G\left(\frac{x}{\omega},t\right)d\omega\right],$$

$$I_3^S(x,t) = \int_x^1 \frac{d\omega}{w}\left[P_{qq}(x)F_2^{NS}\left(\frac{x}{\omega},t\right) + P_{qg}(x)G\left(\frac{x}{w},t\right)\right].$$

The strong coupling constant $\alpha_S(Q^2)$ [26], is related with the β-function by

$$\beta(\alpha_S) = \frac{\partial\alpha_S(Q^2)}{\partial\log Q^2} = -\frac{\beta_0}{4\pi}\alpha_S^2 - \frac{\beta_1}{16\pi^2}\alpha_S^3 - \frac{\beta_2}{64\pi^3}\alpha_S^4 + \cdots$$

where, $\beta_0 = \frac{11}{3}N_C - \frac{4}{3}T_f$, $\beta_1 = \frac{34}{3}N_C^2 - \frac{10}{3}N_C N_f - 2C_F N_f = \frac{306-38N_f}{3}$ and $\beta_2 = \frac{2857}{54}N_C^3 + 2C_F^2 T_f - \frac{205}{9}C_F N_C T_f + \frac{44}{9}C_F T_f^2 + \frac{158}{27}N_C T_f^2 = \frac{2857}{2} - \frac{6673}{18}N_f + \frac{325}{54}N_f^2$ are the one-loop, two-loop and three-loop corrections to the QCD β-function and N_f being the flavor number. We have set $N_C = 3$, $C_F = \frac{N_C^2-1}{2N_C} = \frac{4}{3}$ and $T_f = \frac{1}{2}N_f$.

Now using Taylor expansion and retaining only first two terms we get

$$F_2^S\left(\frac{x}{\omega},t\right) = F_2^S(x,t) + \frac{xu}{1-u}\frac{\partial F_2^S(x,t)}{\partial x}, \quad (2)$$

$$G\left(\frac{x}{\omega},t\right) = G(x,t) + \frac{xu}{1-u}\frac{\partial G(x,t)}{\partial x}, \quad (3)$$

where $u = 1-\omega$. Here we have neglected the terms containing x^2 and higher powers of x since x is small ($<<1$) in our region of discussion. In order to solve Eq. (1), we need to relate the singlet structure function $F_2^S(x,t)$ with the gluon structure function$G(x,t)$. For simplicity, we can assume $G(x,t) = K(x)F_2^S(x,t)$, where $K(x)$ is a suitable function of x or may be a constant. The consideration of this relation is based on the assumptions that the t-evolution of singlet and gluon distribution are identical [27]. Other reason for the use of this relation is that the input singlet and gluon parameterizations, taken for the global analysis to constitute different high precision data, are also functions of x at fixed t [28]. However, the actual functional form of $K(x)$ can be determined by simultaneous solutions of coupled equations of singlet and gluon structure functions. But one can also use a more general version of the relation $G(x,t) = K(x,t)F_2^S(x,t)$ which is close to reality and compatible with the momentum sum rule.

Using Eqs. (2) and (3) in Eq. (1) and performing u integrations we get

$$-t\frac{\partial F_2^S(x,t)}{\partial t} + L(x,t)\frac{\partial F_2^S(x,t)}{\partial x} + M(x,t)F_2^S(x,t) = 0, \quad (4)$$

where,

$$L(x) = \frac{3}{2}A_f[(A_2+KA_4) + T_0(B_2+KC_4) + T_1(C_2+KC_4)]$$

and

$$M(x) = \frac{3}{2}A_f\left[\left(A_1+KA_3+\frac{\partial K}{\partial x}A_4\right) + T_0\left(B_1+KB_3+\frac{\partial K}{\partial x}B_4\right) + T_1\left(C_1+KC_3+\frac{\partial K}{\partial x}C_4\right)\right]$$

with $\frac{\alpha_S(t)}{2\pi} = \frac{3A_f}{2t} = T(t)$.

To linearise $T^2(t)$ and $T^3(t)$ we consider two suitable numerical parameters T_0 and T_1 such that $T^2(t) = T_0\cdot T(t)$ and $T^3(t) = T_1\cdot T(t)$ [17].

The general solution of Eq. (4) is $F(U,V) = 0$, where $F(U,V)$ is an arbitrary function. Here $U(x,t,F_2^S) = K_1$ and $V(x,t,F_2^S) = K_2$ are two independent solutions of the Lagrange's equation

$$\frac{\partial x}{L(x,t)} = \frac{\partial t}{-t} = \frac{\partial F_2^S(x,t)}{-M(x,t)F_2^S(x,t)}. \quad (5)$$

To get the solution of Eq. (5), we will introduce functions $\overline{L}(x)$and $\overline{M}(x)$ such that $L(x,t) = \frac{\beta_0 t}{2}T(t)\overline{L}(x)$ and $M(x,t) = \frac{\beta_0 t}{2}T(t)\,\overline{M}(x)$ [17]. Thus solving Eq. (5) we obtain

$$F_2^S(x,t) = -\gamma\, t^{1+\left(\frac{b-b^2}{t}\right)}\exp\left(\frac{b-c-b^2\ln^2 t}{t}\right)\exp\left[\int\left(\frac{1}{\overline{L}(x)} - \frac{\overline{M}(x)}{\overline{L}(x)}\right)dx\right], \quad (6)$$

where $\gamma = \frac{\alpha}{\beta}$, $b = \frac{\beta_1}{\beta_0^2}$ and $c = \frac{\beta_2}{\beta_0^3}$ are some constants.

Now defining an input point$F_2^S(x,t_0)$at $t = t_0$, where $t_0 = \ln\left(Q^2/\Lambda^2\right)$ for any lower value $Q^2 = Q_0^2$, we get the

t-evolution of deuteron structure function $F_2^d(x,t)$ in NNLO from Eq. (6) as

$$F_2^d(x,t) = F_2^d(x,t_0)\left(\frac{t^{1+\left(b-b^2\right)}}{t_0^{1+(b-b^2)/t_0}}\right) \exp\left(\frac{b-c-b^2\ln^2 t}{t} - \frac{b-c-b^2\ln^2 t_0}{t_0}\right) \tag{7}$$

where, $F_2^d(x,t) = \frac{5}{9}F_2^S(x,t)$.

Similarly defining $F_2^S(x_0,t)$ at $x = x_0$, the x-evolution of deuteron structure function in NNLO is obtained from Eq. (6) as

$$F_2^d(x,t) = F_2^d(x_0,t)\exp\left[\int_{x_0}^{x}\left(\frac{1}{\bar{L}(x)} - \frac{\bar{M}(x)}{\bar{L}(x)}\right)dx\right]. \tag{8}$$

Eqs. (7) and (8) are used in our phenomenological work for calculation of deuteron structure function.

3. Results and discussion

We have compared our results of t and x-evolutions of deuteron structure function $F_2^d(x,t)$ with E665 and NMC

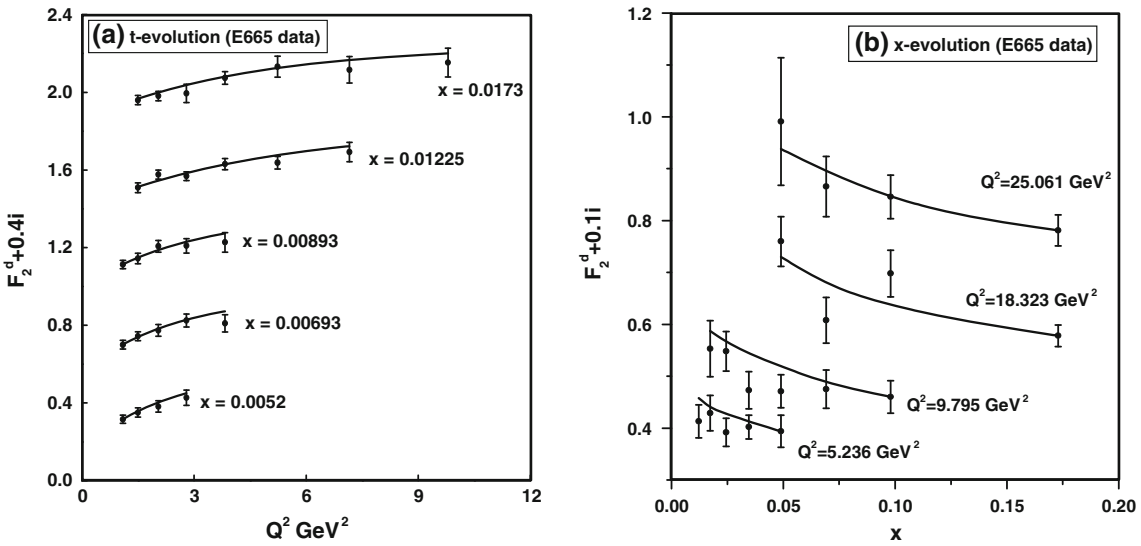

Fig. 1 t and x-evolutions of deuteron structure function compared to E665 data. For clarity, data are scaled up by +0.4i in (**a**) and +0.1i in (**b**) (with i = 0, 1, 2, 3) starting from the bottom of all graphs in each figure

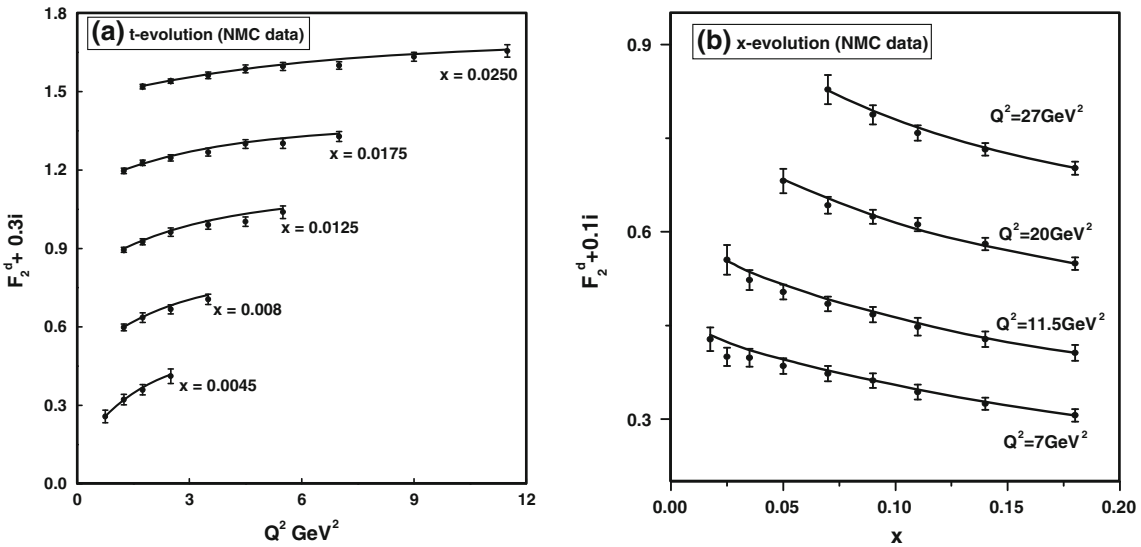

Fig. 2 t and x-evolutions of deuteron structure function compared to NMC data. For clarity, data are scaled up by +0.2i in (**a**) and +i in (**b**) (with i = 0, 1, 2, 3) starting from the bottom of all graphs in each figure

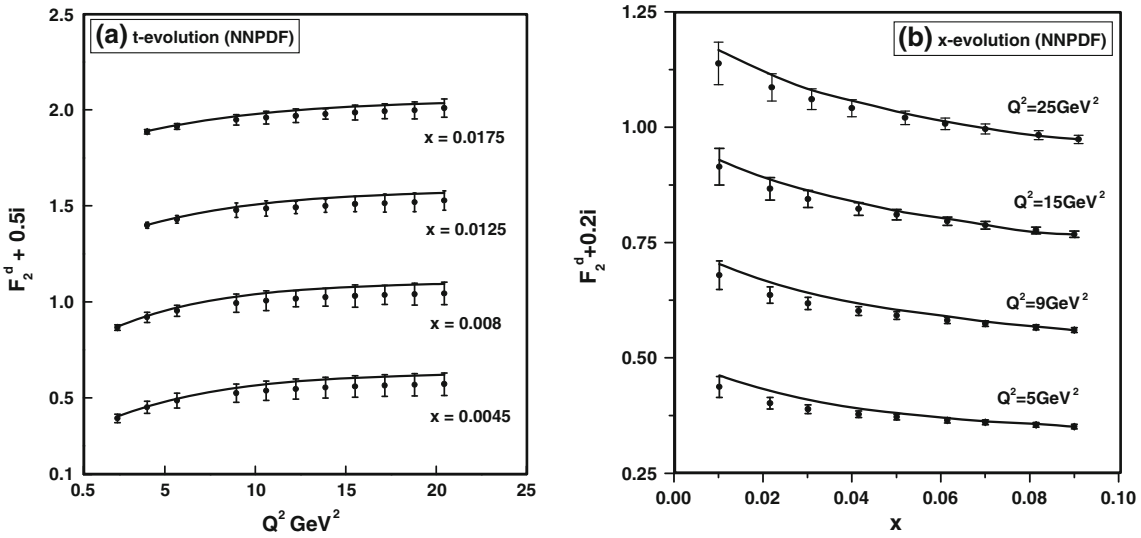

Fig. 3 t and x-evolutions of deuteron structure function compared to NNPDF parametrization. For clarity, data are scaled up by $+0.4$i in (**a**) and $+0.1$i in (**b**) (with i = 0, 1, 2, 3) starting from the bottom of all graphs in each figure

experimental data, and NNPDF parameterizations in Figs. 1a, b; 2a, b and Fig. 3a, b respectively. We consider the range $0.0052 \leq x \leq 0.18$ and $1.094 \leq Q^2 \leq 26\ GeV^2$ for E665 data, $0.0045 \leq x \leq 0.18$ and $0.75 \leq Q^2 \leq 27\ GeV^2$ for NMC data and $0.0045 \leq x \leq 0.09$ and $1.25 \leq Q^2 \leq 26\ GeV^2$ for NNPDF parametrization. In the t-evolutions of deuteron structure function our computed values of $F_2^d(x,t)$ from Eq. (7) are plotted against Q^2 for different values of x. On the other hand, for x- evolutions our computed values of $F_2^d(x,t)$ from Eq. (8) are plotted against x for different values of Q^2. Here vertical error bars are total statistical and systematic uncertainties. In all graphs, the lowest-Q^2 and highest-x points are taken as inputs for $F_2^d(x,t_0)$ and $F_2^d(x_0,t)$ respectively.

4. Conclusions

We have observed that the t and x-evolutions of deuteron structure function are in good consistency with the experimental data and parametrization. Though various methods like Laguerre polynomials, brute-force methods, Mellin transformation etc. are available in order to obtain a numerical solution of DGLAP evolution equations, but our method to solve these equations analytically is also a viable alternative. It changes the integro-differential DGLAP equation into first order partial differential equations which are easier to solve. Here we have considered very few numbers of parameters compared to other methods. Moreover, with this method we have calculated the x-evolution of deuteron structure function.

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